

Basics of Computers - Number System

The technique to represent and work with numbers is called **number system**. **Decimal number system** is the most common number system. Other popular number systems include **binary number system**, **octal number system**, **hexadecimal number system**, etc.

There are four number systems that a computer supports. They are –

1. Binary
2. Octal
3. Decimal
4. Hexadecimal

S.No.	Number System	Description	Example
1	Binary Number System	<ul style="list-style-type: none">• Base 2.• Digits used: 0, 1	11010
2	Octal Number System	<ul style="list-style-type: none">• Base 8.• Digits used: 0 to 7	125708
3	Hexadecimal Number System	<ul style="list-style-type: none">• Base 16.• Digits used: 0 to 9• Letters used: A- F	27FB
4	Decimal Number System	<ul style="list-style-type: none">• Base 10.• Digits used: 0 to 9	1234

Decimal Number System

Decimal number system is a **base 10** number system having 10 digits from 0 to 9. This means that any numerical quantity can be represented using these 10 digits. Decimal number system is also a **positional value system**. This means that the value of digits will depend on its position. Let us take an example to understand this.

Say we have three numbers – 734, 971 and 207. The value of 7 in all three numbers is different–

- In 734, value of 7 is 7 hundreds or 700 or 7×100 or 7×10^2
- In 971, value of 7 is 7 tens or 70 or 7×10 or 7×10^1
- In 207, value of 7 is 7 units or 7 or 7×1 or 7×10^0

The weightage of each position can be represented as follows –

10^5	10^4	10^3	10^2	10^1	10^0
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Octal Number System

Octal number system has eight digits – 0, 1, 2, 3, 4, 5, 6 and 7. Octal number system is also a positional value system with where each digit has its value expressed in powers of 8, as shown here –

8^5	8^4	8^3	8^2	8^1	8^0
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Decimal equivalent of any octal number is sum of product of each digit with its positional value.

$$\begin{aligned}726_8 &= 7 \times 8^2 + 2 \times 8^1 + 6 \times 8^0 \\ &= 448 + 16 + 6 \\ &= 470_{10}\end{aligned}$$

Hexadecimal Number System

Hexadecimal Number System has 16 symbols – 0 to 9 and A to F where A is equal to 10, B is equal to 11 and so on till F. Hexadecimal number system is also a positional value system with where each digit has its value expressed in powers of 16, as shown here –

16^5	16^4	16^3	16^2	16^1	16^0
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Decimal equivalent of any hexadecimal number is sum of product of each digit with its positional value.

$$\begin{aligned}27FB_{16} &= 2 \times 16^3 + 7 \times 16^2 + 15 \times 16^1 + 10 \times 16^0 \\ &= 8192 + 1792 + 240 + 10 \\ &= 10234_{10}\end{aligned}$$

Number System Relationship

The following table depicts the relationship between decimal, binary, octal and hexadecimal number systems.

HEXADECIMAL	DECIMAL	OCTAL	BINARY
0	0	0	0000
1	1	1	0001

2	2	2	0010
3	3	3	0011
4	4	4	0100
5	5	5	0101
6	6	6	0110
7	7	7	0111
8	8	10	1000
9	9	11	1001
A	10	12	1010
B	11	13	1011
C	12	14	1100
D	13	15	1101
E	14	16	1110
F	15	17	1111

Number System Conversion

Decimal to Binary

Decimal numbers can be converted to binary by repeated division of the number by 2 while recording the remainder. Let's take an example to see how this happens.

		Remainder	
2	43		
2	21	1	MSB ↑ LSB
2	10	1	
2	5	0	
2	2	1	
2	1	0	
	0	1	

The remainders are to be read from bottom to top to obtain the binary equivalent.

$$43_{10} = 101011_2$$

Decimal to Octal

Decimal numbers can be converted to octal by repeated division of the number by 8 while recording the remainder. Let's take an example to see how this happens.

		Remainder	
8	473		
8	59	1	MSD ↑ LSD
8	7	3	
	0	7	

Reading the remainders from bottom to top,

$$473_{10} = 731_8$$

Decimal to Hexadecimal

Decimal numbers can be converted to octal by repeated division of the number by 16 while recording the remainder. Let's take an example to see how this happens.

		Remainder	
16	423		
16	26	7	↑
16	1	A	
	0	1	

Reading the remainders from bottom to top we get,

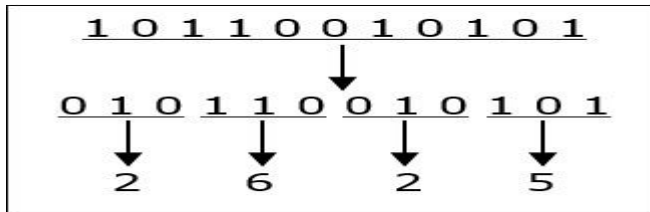
$$423_{10} = 1A7_{16}$$

Binary to Octal

To convert a binary number to octal number, these steps are followed –

- Starting from the least significant bit, make groups of three bits.
- If there are one or two bits less in making the groups, 0s can be added after the most significant bit
- Convert each group into its equivalent octal number

Let's take an example to understand this.



$$10110010101_2 = 2625_8$$

Binary to Octal conversion Examples

Example 1: Convert 1010101_2 to octal

Solution:

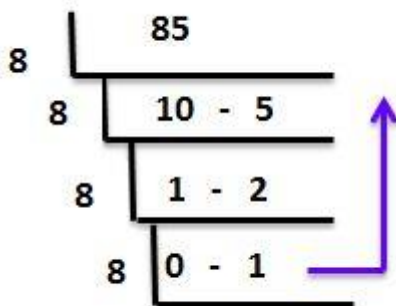
Given binary number are 1010101_2

First, we convert given binary to decimal

$$\begin{aligned} 1010101_2 &= (1 * 2^6) + (0 * 2^5) + (1 * 2^4) + (0 * 2^3) + (1 * 2^2) + (0 * 2^1) + (1 * 2^0) \\ &= 64 + 0 + 16 + 0 + 4 + 0 + 1 \\ &= 64 + 21 \end{aligned}$$

$$1010101_2 = 85 \text{ (Decimal form)}$$

Now we will convert this decimal to octal form



Therefore, the equivalent octal number is 125_8 .

Example 2: Convert 01101_2 to octal

Solution:

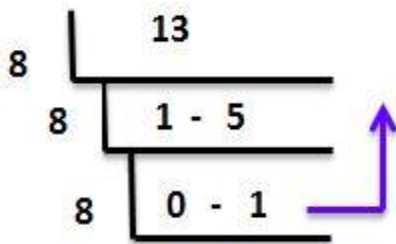
Given binary number is 01101_2

First we convert given binary to decimal

$$\begin{aligned} 01101_2 &= (0 * 2^4) + (1 * 2^3) + (1 * 2^2) + (0 * 2^1) + (1 * 2^0) \\ &= 0 + 8 + 4 + 0 + 1 \end{aligned}$$

$$01101_2 = 13 \text{ (Decimal form)}$$

Now we will convert this decimal to octal form



Therefore, the equivalent octal number is 15_8 .

To convert an octal number to binary, each octal digit is converted to its 3-bit binary equivalent according to this table.

Octal Digit	0	1	2	3	4	5	6	7
Binary Equivalent	000	001	010	011	100	101	110	111

$$54673_8 = 101100110111011_2$$

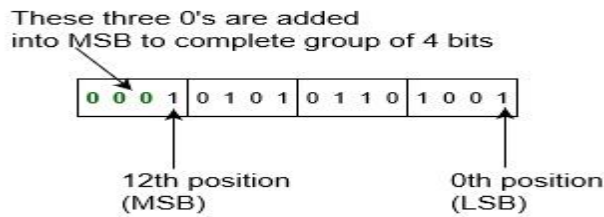
Binary to Hexadecimal

To convert a binary number to hexadecimal number, these steps are followed –

- Starting from the least significant bit, make groups of four bits.
- If there are one or two bits less in making the groups, 0s can be added after the most significant bit.
- Convert each group into its equivalent octal number.

Let's take an example to understand this.

Example-1 – Convert binary number 1010101101001 into hexadecimal number. Since there is no binary point here and no fractional part. So,



Therefore, Binary to hexadecimal is,

$$\begin{aligned}
 &= (1010101101001)_2 \\
 &= (1\ 0101\ 0110\ 1001)_2 \\
 &= (0001\ 0101\ 0110\ 1001)_2 \\
 &= (1\ 5\ 6\ 9)_{16} \\
 &= (1569)_{16}
 \end{aligned}$$

Hexadecimal to Binary Examples

Question 1: Convert $A2B_{16}$ to an equivalent binary number.

Solution: Given hexadecimal number = $A2B_{16}$

First, convert the given hexadecimal to the equivalent decimal number.

$$\begin{aligned}
 A2B_{16} &= (A \times 16^2) + (2 \times 16^1) + (B \times 16^0) \\
 &= (A \times 256) + (2 \times 16) + (B \times 1) \\
 &= (10 \times 256) + 32 + 11 \\
 &= 2560 + 43 \\
 &= 2603(\text{Decimal number})
 \end{aligned}$$

Now we have to convert 2603_{10} to binary

```

2 | 2603
2 | 1301 -- 1
2 | 650 -- 1
2 | 325 -- 0
2 | 162 -- 1
2 | 81 -- 0
2 | 40 -- 1
2 | 20 -- 0
2 | 10 -- 0
2 | 5 -- 0
2 | 2 -- 1
2 | 1 -- 0
2 | 0 -- 1
    
```

The binary number obtained is 101000101011_2

Hence, $A2B_{16} = 101000101011_2$

Hexadecimal to Decimal:

Example 3: Convert 121_{16} to decimal number.

$$\text{Solution: } 1 \times 16^2 + 2 \times 16^1 + 1 \times 16^0$$

$$= 16 \times 16 + 2 \times 16 + 1 \times 1$$

$$= 289$$

Therefore, $121_{16} = 289_{10}$

Example: Convert $1BC_{16}$ into an octal number.

Solution: Given, $1BC_{16}$ is a hexadecimal number.

$$1 \rightarrow 0001, B \rightarrow 1011, C \rightarrow 1100$$

Now group them from right to left, each having 3 digits.

$$000, 110, 111, 100$$

$$000 \rightarrow 0, 110 \rightarrow 6, 111 \rightarrow 7, 100 \rightarrow 4$$

$$\text{Hence, } 1BC_{16} = 674_8$$

Hexadecimal to Octal Questions

Q.1: Find the equivalent octal form of $C1_{16}$.

Solution: Given, a hexadecimal number is $C1$

$$C1_{16} = (C \times 16^1) + (1 \times 16^0)$$

$$= C \times 16 + 1 \times 1$$

$$= 12 \times 16 + 1$$

$$= 192 + 1$$

$$C1_{16} = 193 \text{ (Decimal form)}$$

Now we have to convert this decimal to octal number;

$$8 \overline{)193}$$

$$8 \overline{)24} \text{ -- } 1$$

$$8 \overline{)3} \text{ -- } 0$$

$$8 \overline{)0} \text{ -- } 3$$

The octal number is 301_8

$$\text{Hence, } C1_{16} = 301_8$$

Q.2: Find the equivalent octal form of F_{16} .

Solution: Given, a hexadecimal number is F.

$$F_{16} = (F \times 16^0)$$

$$= F \times 1$$

$$= F$$

$$= 15(\text{Decimal form})$$

Now we have to convert this decimal to equivalent octal number;

$$\begin{array}{r} 8 \overline{)15} \\ 8 \overline{)1} \text{ -- } 7 \\ 8 \overline{)0} \text{ -- } 1 \end{array}$$

The octal number is 17_8

$$\text{Hence, } F_{16} = 17_8$$

Q.3: Find the equivalent octal form of 105_{16}

Solution: Given, a hexadecimal number is 105.

$$105_{16} = (1 \times 16^2) + (0 \times 16^1) + (5 \times 16^0)$$

$$= 1 \times 256 + 0 \times 16 + 5 \times 1$$

$$= 256 + 0 + 5$$

$$= 261(\text{Decimal form})$$

Now we have to convert this decimal to equivalent octal;

$$\begin{array}{r} 8 \overline{)261} \\ 8 \overline{)32} \text{ -- } 5 \\ 8 \overline{)4} \text{ -- } 0 \\ 8 \overline{)0} \text{ -- } 4 \end{array}$$

The octal number is 405_8

$$\text{Hence, } 105_{16} = 405_8$$

OCTAL TO BINARY

Q.1: Convert 41_8 to a binary number.

Solution: Given number is 41_8

$$41_8 = (4 * 8^1) + (1 * 8^0)$$

$$= 4 * 8 + 1 * 1$$

$$= 32 + 1$$

$$= 33(\text{Decimal number})$$

Now convert this decimal number into its equivalent binary number. Let us draw a table to show the conversion of decimal to binary as given below.

Decimal Number divided by 2	Quotient	Remainder
33 divided by 2	16	1
16 divided by 2	8	0
8 divided by 2	4	0
4 divided by 2	2	0
2 divided by 2	1	0
1 divided by 2	0	1

Therefore, the equivalent binary number is 100001_2 .

Hence, $41_8 = 100001_2$

Convert 10_8 to a binary number.

Solution: Given number is 10_8

$$10_8 = (1 * 8^1) + (0 * 8^0)$$

$$= 1 * 8 + 0 * 1$$

$$= 8 + 0$$

$$= 8 \text{ (Decimal number)}$$

Now convert this decimal number into its equivalent binary number. Let us draw a table to show the conversion of decimal to binary as given below.

Decimal Number divided by 2	Quotient	Remainder
8 divided by 2	4	0
4 divided by 2	2	0
2 divided by 2	1	0
1 divided by 2	0	1

Therefore, the equivalent binary number is 1000_2 .

Hence, $8_8 = 1000_2$

Octal to Decimal

To convert an octal number to a decimal number we need to multiply each digit of the given octal with the reducing power of 8.

Let us learn here, the conversion of Octal number to Decimal Number or base 8 to base 10.

Examples on Octal to Decimal

Example 1: Suppose 215_8 is an octal number, then it's decimal form will be,

$$\begin{aligned}215_8 &= 2 \times 8^2 + 1 \times 8^1 + 5 \times 8^0 \\ &= 2 \times 64 + 1 \times 8 + 5 \times 1 = 128 + 8 + 5 \\ &= 141_{10}\end{aligned}$$

Example 2: Let 125 is an octal number denoted by 125_8 . Find the decimal number.

$$\begin{aligned}125_8 &= 1 \times 8^2 + 2 \times 8^1 + 5 \times 8^0 \\ &= 1 \times 64 + 2 \times 8 + 5 \times 1 = 64 + 16 + 5 \\ &= 85_{10}\end{aligned}$$

Octal to Hexadecimal Number

Hexadecimal numbers consist of numbers and alphabets. It is represented with base 16. The numbers from 0-9 are represented in the usual form, but from 10 to 15, it is denoted as A, B, C, D, E, F. Conversion of the octal number to hexadecimal requires two steps.

- First, convert octal numbers to decimal numbers.
- Then, convert decimal numbers to hexadecimal numbers.

Example

Let us understand with the help of an example. We will take the same example, where we have converted the octal numbers to decimal, such as;

$$(55)_8 = (45)_{10}$$

Now, convert $(45)_{10}$ into a hexadecimal number by dividing 45 by 16 until you get a remainder less than 16.

$$\begin{array}{r|l} 16 & 45 \\ \hline 16 & 2 \text{ ---} 13 \\ \hline & 0 \text{ ---} 2 \end{array}$$

Therefore, we can write, $(45)_{10} = (2D)_{16}$

Or $(55)_8 = (2D)_{16}$

Hexa To Binary Examples

Question 1: Convert $A2B_{16}$ to an equivalent binary number.

Solution: Given hexadecimal number = $A2B_{16}$

First, convert the given hexadecimal to the equivalent decimal number.

$$\begin{aligned}A2B_{16} &= (A \times 16^2) + (2 \times 16^1) + (B \times 16^0) \\ &= (A \times 256) + (2 \times 16) + (B \times 1) \\ &= (10 \times 256) + 32 + 11\end{aligned}$$

$$= 2560 + 43$$

$$= 2603(\text{Decimal number})$$

Now we have to convert 2603_{10} to binary

$$\begin{array}{r} 2 \mid 2603 \\ 2 \mid 1301 \text{ -- } 1 \\ 2 \mid 650 \text{ -- } 1 \\ 2 \mid 325 \text{ -- } 0 \\ 2 \mid 162 \text{ -- } 1 \\ 2 \mid 81 \text{ -- } 0 \\ 2 \mid 40 \text{ -- } 1 \\ 2 \mid 20 \text{ -- } 0 \\ 2 \mid 10 \text{ -- } 0 \\ 2 \mid 5 \text{ -- } 0 \\ 2 \mid 2 \text{ -- } 1 \\ 2 \mid 1 \text{ -- } 0 \\ 2 \mid 0 \text{ -- } 1 \end{array}$$

The binary number obtained is 101000101011_2

$$\text{Hence, } A2B_{16} = 101000101011_2$$

Question 2: Convert E_{16} to an equivalent binary number.

Solution: Given, a hexadecimal number is E.

First, convert the given hexadecimal to the equivalent decimal number.

$$E_{16} = E \times 16^0$$

$$= E \times 1$$

$$= E$$

$$= 14 (\text{Decimal number})$$

Now we have to convert 14_{10} to binary number.

$$\begin{array}{r} 2 \mid 14 \\ 2 \mid 7 \text{ -- } 0 \\ 2 \mid 3 \text{ -- } 1 \\ 2 \mid 1 \text{ -- } 1 \\ 2 \mid 0 \text{ -- } 1 \end{array}$$

The binary number obtained is 1110_2

$$\text{Hence, } E_{16} = 1110_2$$

Question 3: Convert 30_{16} to an equivalent binary number.

Solution: Given the hexadecimal number is 30

First, convert the given hexadecimal to the equivalent decimal number.

$$30_{16} = (3 \times 16^1) + (0 \times 16^0)$$

$$= 48 + 0$$

$$= 48(\text{Decimal number})$$

Now we have to convert 48_{10} to binary.

Therefore,

```

2 | 48
2 | 24 -- 0
2 | 12 -- 0
2 | 6 -- 0
2 | 3 -- 0
2 | 1 -- 1
2 | 0 -- 1

```

The binary number is 110000_2

Hence, $30_{16} = 110000_2$

Hex to Decimal Converter

Example 1:

Convert $(1DA6)_{16}$ to decimal.

Solution:

$(1DA6)_{16}$

Here,

$1 = 1$

$D = 13$

$A = 10$

$6 = 6$

Thus,

$$\begin{aligned}
 (1DA6)_{16} &= (1 \times 16^3) + (13 \times 16^2) + (10 \times 16^1) + (6 \times 16^0) \\
 &= (1 \times 4096) + (13 \times 256) + (10 \times 16) + (6 \times 1) \\
 &= 4096 + 3328 + 160 + 6 \\
 &= 7590
 \end{aligned}$$

Therefore, $(1DA6)_{16} = (7590)_{10}$

Example 2:

Convert $(E8B)_{16}$ to decimal system.

Solution:

$(E8B)_{16}$

Here,

$E = 14$

$8 = 8$

$B = 11$

Thus,

$$\begin{aligned}
 (E8B)_{16} &= (14 \times 16^2) + (8 \times 16^1) + (11 \times 16^0) \\
 &= (14 \times 256) + (8 \times 16) + (11 \times 1) \\
 &= 3584 + 128 + 11 \\
 &= 3723
 \end{aligned}$$

Therefore, $(E8B)_{16} = (3723)_{10}$

Convert $1BC_{16}$ into an octal number.

Solution: Given, $1BC_{16}$ is a hexadecimal number.

$1 \rightarrow 0001$, $B \rightarrow 1011$, $C \rightarrow 1100$

Now group them from right to left, each having 3 digits.

000, 110, 111, 100

000→0, 110 →6, 111→7, 100→4

Hence, $1BC_{16} = 674_8$

Hexadecimal to Octal Questions

Q.1: Find the equivalent octal form of $C1_{16}$.

Solution: Given, a hexadecimal number is C1

$$C1_{16} = (C \times 16^1) + (1 \times 16^0)$$

$$= C \times 16 + 1 \times 1$$

$$= 12 \times 16 + 1$$

$$= 192 + 1$$

$$C1_{16} = 193 \text{ (Decimal form)}$$

Now we have to convert this decimal to octal number;

$$\begin{array}{r} 8 \overline{)193} \\ 8 \overline{)24} \text{ -- } 1 \\ 8 \overline{)3} \text{ -- } 0 \\ 8 \overline{)0} \text{ -- } 3 \end{array}$$

The octal number is 301_8

Hence, $C1_{16} = 301_8$

Q.2: Find the equivalent octal form of F_{16} .

Solution: Given, a hexadecimal number is F.

$$F_{16} = (F \times 16^0)$$

$$= F \times 1$$

$$= F$$

$$= 15 \text{ (Decimal form)}$$

Now we have to convert this decimal to equivalent octal number;

$$\begin{array}{r} 8 \overline{)15} \\ 8 \overline{)1} \text{ -- } 7 \\ 8 \overline{)0} \text{ -- } 1 \end{array}$$

The octal number is 17_8

Hence, $F_{16} = 17_8$

Binary addition

Binary addition is one of the binary operations. To recall, the term “Binary Operation” represents the basic operations of mathematics that are performed on two operands.

Rules of Binary Addition

Binary addition is much easier than the decimal addition when you remember the following tricks or rules. Using these rules, any binary number can be easily added. The four rules of binary addition are:

Case	A + B	Sum	Carry
1	0 + 0	0	0
2	0 + 1	1	0
3	1 + 0	1	0
4	1 + 1	0	1

Examples of Binary Addition

A few examples of binary additions are as follows:

Example 1: 10001 + 11101

Solution:

$$\begin{array}{r}
 1 \\
 10001 \\
 (+) 11101 \\
 \hline
 101110
 \end{array}$$

Example 2: 10111 + 110001

Solution:

$$\begin{array}{r}
 111 \\
 10111 \\
 (+) 110001 \\
 \hline
 1001000
 \end{array}$$

Binary subtraction

Binary subtraction is one of the four binary operations, where we perform the subtraction method for two binary numbers (comprising only two digits, 0 and 1). This operation is similar to the basic arithmetic subtraction performed on decimal numbers in Maths. Hence, when we subtract 1 from 0, we need to borrow 1 from the next higher order digit, to reduce the digit by 1 and the remainder left here is also 1. Read other binary operations here.

Case	A - B	Subtract	Borrow
1	0 - 0	0	0
2	1 - 0	1	0
3	1 - 1	0	0
4	0 - 1	0	1

Binary Subtraction Examples

Consider other examples of binary subtractions are as follows:

Example 1: 0011010 – 001100

Solution:

$$\begin{array}{r}
 11 \text{ Borrow} \\
 0011010 \\
 (-) 001100 \\
 \hline
 \end{array}$$

0 0 0 1 1 1 0

Decimal Equivalent:

0 0 1 1 0 1 0 = 26

0 0 1 1 0 0 = 12

Therefore, 26 – 12 = 14

The binary resultant 0 0 0 1 1 1 0 is equivalent to 14.

Example 2: 0100010 – 0001010

Solution:

1 1 Borrow

0 1 0 0 0 1 0 = 34₁₀

(-) 0 0 0 1 0 1 0 = 10₁₀

0 0 1 1 0 0 0 = 24₁₀

Binary multiplication

Binary multiplication is one of the four binary arithmetic. The other three fundamental operations are addition, subtraction and division. In the case of a binary operation, we deal with only two digits, i.e. 0 and 1. The operation performed while finding the binary product is similar to the conventional multiplication method. The four major steps in binary digit multiplication are:

Case	A	x	B	Multiplication
1	0	x	0	0
2	0	x	1	0
3	1	x	0	0
4	1	x	1	1

Examples of Binary Multiplication

Some binary multiplication examples are given below for a better understanding of this concept.

Example 1: Solve 1010 × 101

Solution:

1010 × 101

1010

(×) 101

1010

0000

01010 First Intermediate Sum

1010

110010

Comparison with Decimal values:

$$1010_2 = 10_{10}$$

$$1010_2 = 5_{10}$$

$$10 \times 5 = 50_{10}$$

$$(110010)_2 = 50_{10}$$

Another example of binary multiplication with a decimal point is as follows:

Question: 1011.01×110.1

Solution:

$$\begin{array}{r} 1011.01 \\ 110.1 \\ \hline 101101 \\ 000000 \\ \hline 0101101 \text{ First Intermediate sum} \\ 101101 \\ \hline 11100001 \text{ Second Intermediate Sum} \\ 101101 \\ \hline 1001001.001 \text{ Final Sum} \end{array}$$

Here, the decimal point is placed three places from the least significant bit. Because, the binary number 1011.01, the decimal point is 2 places from the LSB and 110.1 the decimal point is placed 1 place from the LSB.

What is Binary Division?

The binary division operation is similar to the base 10 decimal system, except the base 2. The division is probably one of the most challenging operations of the basic arithmetic operations. There are different ways to solve division problems using binary operations. Long division is one of them and the easiest and the most efficient way.

Binary Division Rules

The binary division is much easier than the decimal division when you remember the following division rules. The main rules of the binary division include:

- $1 \div 1 = 1$
- $1 \div 0 = \text{Meaningless}$
- $0 \div 1 = 0$
- $0 \div 0 = \text{Meaningless}$

Binary Division Examples

Example 1.

Question: Solve $01111100 \div 0010$

Solution:

Given

$01111100 \div 0010$

Here the dividend is 01111100 , and the divisor is 0010

Remove the zero's in the **Most Significant Bit** in both the dividend and divisor, that doesn't change the value of the number.

So the dividend becomes 1111100 , and the divisor becomes 10 .

Now, use the long division method.

$$\begin{array}{r} 10 \) \ 1111100 \quad (111110 \\ \underline{(-) \ 10} \\ 11 \\ \underline{(-) \ 10} \\ 11 \\ \underline{(-) \ 10} \\ 11 \\ \underline{(-) \ 10} \\ 10 \\ \underline{(-) \ 10} \\ 00 \\ \underline{ } \\ 00 \end{array}$$

Example 2: Solve using the long division method: $101101 \div 101$

Solution:

$$\begin{array}{r}
 101 \overline{) 101101} \quad (1001) \\
 \underline{(-) 101} \\
 101 \\
 \underline{(-) 101} \\
 0
 \end{array}$$

Complement Arithmetic

Complements are used in the digital computers in order to simplify the subtraction operation and for the logical manipulations. For each radix-r system (radix r represents base of number system) there are two types of complements.

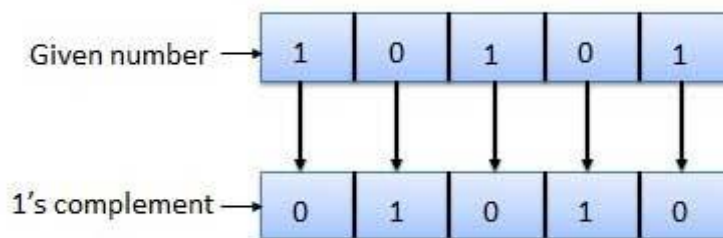
S.N.	Complement	Description
1	Radix Complement	The radix complement is referred to as the r's complement
2	Diminished Radix Complement	The diminished radix complement is referred to as the (r-1)'s complement

Binary system complements

As the binary system has base $r = 2$. So the two types of complements for the binary system are 2's complement and 1's complement.

1's complement

The 1's complement of a number is found by changing all 1's to 0's and all 0's to 1's. This is called as taking complement or 1's complement. Example of 1's Complement is as follows.

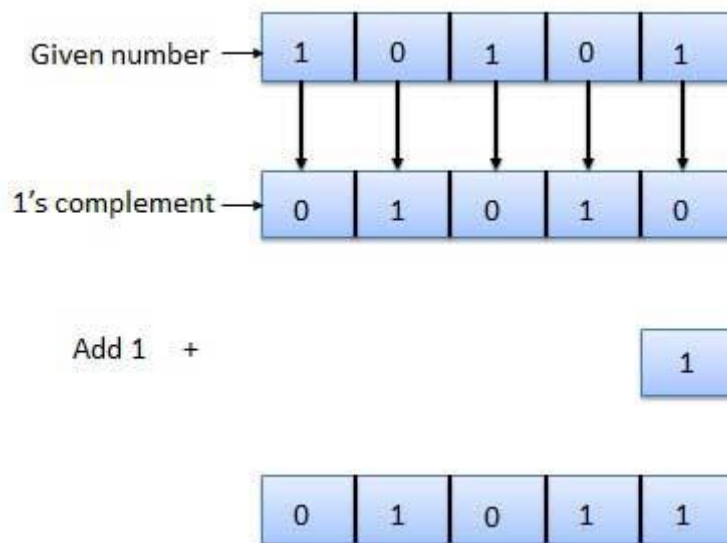


2's complement

The 2's complement of binary number is obtained by adding 1 to the Least Significant Bit (LSB) of 1's complement of the number.

2's complement = 1's complement + 1

Example of 2's Complement is as follows.



9's and 10's Complement

If the number is binary, then we use 1's complement and 2's complement. But in case, when the number is a decimal number, we will use the 9's and 10's complement. The 10's complement is obtained from the 9's complement of the number, and we can also find the 9's and 10's complement using the r 's and $(r-1)$'s complement formula.

9's Complement

The 9's complement is used to find the subtraction of the decimal numbers. The 9's complement of a number is calculated by subtracting each digit of the number by 9. For example, suppose we have a number 1423, and we want to find the 9's complement of the number. For this, we subtract each digit of the number 1423 by 9. So, the 9's complement of the number 1423 is $9999 - 1423 = 8576$.

10's Complement

The 10's complement is also used to find the subtraction of the decimal numbers. The 10's complement of a number is calculated by subtracting each digit by 9 and then adding 1 to the result. Simply, by adding 1 to its 9's complement we can get its 10's complement value. For example, suppose we have a number 1423, and we want to find the 10's complement of the number. For this, we find the 9's complement of the number 1423 that is $9999 - 1423 = 8576$, and now we will add 1 to the result. So the 10's complement of the number 1423 is $8576 + 1 = 8577$.

Radix and Diminished Radix complement

The mostly used complements are 1's, 2's, 9's, and 10's complement. Apart from these complements, there are many more complements from which mostly peoples are not familiar. For finding the subtraction of the number base system, the complements are used. If r is the base of the number system, then there are two types of complements that are possible, i.e., r 's and $(r-1)$'s. We can find the r 's complement, and $(r-1)$'s complement of the number, here r is the radix. The r 's complement is also known as **Radix complement** $(r-1)$'s complement, is known as **Diminished Radix complement**.

If the base of the number is 2, then we can find 1's and 2's complement of the number. Similarly, if the number is the octal number, then we can find 7's and 8's complement of the number.

Example 1: $(1011000)_2$

This number has a base of 2, which means it is a binary number. So, for the binary numbers, the value of r is 2, and $r-1$ is $2-1=1$. So, we can calculate the 1's and 2's complement of the number.

1's complement of the number 1011000 is calculated as:

$$\begin{aligned} &= \{(2^7)_{10}-1\}-(1011000)_2 \\ &= \{(128)_{10}-1\}-(1011000)_2 \\ &= \{(127)_{10}\}-(1011000)_2 \\ &= 1111111_2-1011000_2 \\ &= 0100111 \end{aligned}$$

2's complement of the number 1011000 is calculated as:

$$\begin{aligned} &= (2^7)_{10}-(1011000)_2 \\ &= (128)_{10}-(1011000)_2 \\ &= 1000000_2-1011000_2 \\ &= 0101000_2 \end{aligned}$$

Example 2: $(155)_{10}$

This number has a base of 10, which means it is a decimal number. So, for the decimal numbers, the value of r is 10, and $r-1$ is $10-1=9$. So, we can calculate the 10's and 9's complement of the number.

9's complement of the number 155 is calculated as:

$$\begin{aligned} &= \{(10^3)_{10}-1\}-(155)_{10} \\ &= (1000-1)-155 \\ &= 999-155 \\ &= (844)_{10} \end{aligned}$$

10's complement of the number 1011000 is calculated as:

$$\begin{aligned}
&=(10^3)_{10}-(155)_{10} \\
&=1000-155 \\
&=(845)_{10}
\end{aligned}$$

Example 3: (172)₈

This number has a base of 8, which means it is an octal number. So, for the octal numbers, the value of r is 8, and r-1 is 8-1=7. So, we can calculate the 8's and 7's complement of the number.

7's complement of the number 172 is calculated as:

$$\begin{aligned}
&=\{(8^3)_{10}-1\}-(172)_8 \\
&=((512)_{10}-1)-(132)_8 \\
&=(511)_{10}-(122)_{10} \\
&=(389)_{10} \\
&=(605)_8
\end{aligned}$$

8's complement of the number 172 is calculated as:

$$\begin{aligned}
&=(8^3)_{10}-(172)_8 \\
&=(512)_{10}-172_8 \\
&=512_{10}-122_{10} \\
&=390_{10} \\
&=606_8
\end{aligned}$$

Example 4: (F9)₁₆

This number has a base of 16, which means it is a hexadecimal number. So, for the hexadecimal numbers, the value of r is 16, and r-1 is 16-1=15. So, we can calculate the 16's and 15's complement of the number.

15's complement of the number F9 is calculated as:

$$\begin{aligned}
&\{(16^2)_{10}-1\}-(F9)_{16} \\
&(256-1)_{10}-F9_{16} \\
&255_{10}-249_{10} \\
&(6)_{10} \\
&(6)_{16}
\end{aligned}$$

16's complement of the number F9 is calculated as:

$$\begin{aligned}
&\{(16^2)_{10}\}-(F9)_{16} \\
&256_{10}-249_{10} \\
&(7)_{10} \\
&(7)_{16}
\end{aligned}$$